Version 1

Problem 1 Consider the following system :

$$\begin{cases} x + \ \ y - \ \ z = k \\ 2x + 3y + kz = 3k \\ x + ky + 3z = 2k \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & k \\ 2 & 3 & k & 3k \\ 1 & k & 3 & 2k \end{array}\right] \begin{array}{l} r_2 - 2r_1 \to r_2 \\ r_3 - r_1 \to r_3 \end{array} \left[\begin{array}{cccc} 1 & 1 & -1 & k \\ 0 & 1 & k+2 & k \\ 0 & k-1 & 4 & k \end{array}\right] r_3 - (k-1) r_2 \to r_3$$

$$\left[\begin{array}{cccc} 1 & 1 & -1 & k \\ 0 & 1 & k+2 & k \\ 0 & 0 & 4 - (k-1)(k+2) & k - (k-1)k \end{array}\right] \left[\begin{array}{cccc} 1 & 1 & -1 & k \\ 0 & 1 & k+2 & k \\ 0 & 0 & (k-2)(k+3) & k(k-2) \end{array}\right] = A_E$$

1. For which values of k does the system have no solutions?

$k = -3$ (last row of A_E is a "bad row")

2. For which values of k does the system have exactly one solution ?

$k \neq 2, -3$ (First 3 columns of A_E have pivots i.e no free variables)
no bad rows

3. For which values of k does the system have infinitely many solutions ?

$k = 2$ (Last row of A_E is 0 0 0 0
3rd column has no pivot.
there are free variables and
no bad rows)

NAME (First,Last) :

Problem 2 Let A be the 4×4 matrix with columns c_1, c_2, c_3, c_4. The matrix $B = \begin{pmatrix} | & | & | & | & a \\ | & | & | & | & b \\ c_1 & c_2 & c_3 & c_4 & c \\ | & | & | & | & d \end{pmatrix}$

(that is B is the 4×5 matrix that consists of A plus an additional fifth column $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$)

reduces to $\begin{pmatrix} \overbrace{\begin{matrix} 1 & 1 & -1 & 2 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{matrix}}^{C} & \begin{matrix} a \\ b-a \\ c-b+a \\ d-a+2b \end{matrix} \end{pmatrix}$

1. Are c_1, c_2, c_3, and c_4 linearly independent ? Justify your answer.

C is reduced to echelon form and only has 3 pivots, there is no pivot in column 4 so no c_1 c_2 c_3 c_4 are not linearly independent

2. Are c_1, c_2, c_4 linearly independent ? Justify your answer.

yes looking at $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ there is a pivot in every column so c_1 c_2 c_4 are independent.

3. Give an example of a vector $b \in R^4$ that it is not in span (c_1, c_2, c_3, c_4) , or explain why this is not possible. If you give an example, you need to justify why your example works.

we want a vector $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$ with $d-a+2b \neq 0$

For example $b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ would work.

Problem 3 This problem has three unrelated parts.

1. Give an example of a linear system with two equations and three variables that has no solutions, or explain why this is not possible.

$$x_1 + x_2 + x_3 = 1$$
$$x_1 + x_2 + x_3 = 2$$

2. Give an example of a 3x3 matrix A that has linearly independent columns and can be reduced (by performing a sequence of elementary operations) to a matrix B that has linearly dependent columns, or explain why this is not possible.

Since the columns of A are linearly independent $Ax = 0$ has only the solution $x = \vec{0}$, so $Bx = 0$ also has only the solution $x = \vec{0}$, because performing elementery operations transforms a system into a different system with the same solutions. Since $Bx = \vec{0}$ only has the triviel solution, the columns of B are also independent.

IMpossible

3. Give an example of three non zero vectors u_1, u_2, u_3 in R^3 that are linearly dependent, but u_1 is not in span (u_2, u_3), or explain why this is not possible.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$